International Science and Technology Journal
المجلة الدولية للعلوم والتقنية

Volume 36 العدد
Part 1 المجلد

المجلة الدولية للعلوم والتقنية
International Science and Technology Journal
ISTJ

http://www.doi.org/10.62341/nafa3003

Received	**2025/03/02**	**تم استلام الورقة العلمية في**
Accepted	**2025/03/29**	**تم قبول الورقة العلمية في**
Published	**2025/03/29**	**تم نشر الورقة العلمية في**

On $P^* g$- Closed Set in topological Spaces

***Nadiy. A. Altoumi [1], Fatma. A. Toumi[2]**

Department of mathematics, Faculty of Education, University of Zawia
AzZawia - Libya
*Corresponding author
na.altoumi@zu.edu.ly

ABSTRACT

Closed sets play a fundamental role in topological spaces. Notably, a topology on a set can even be characterized by specifying the properties of its closed sets. In 1970, N. Levine introduced the concept of generalized closed sets, defined: A subset $\boldsymbol{S}$ of a topological space X is considered generalized closed if the closure of A is contained in U, $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set. In this study, we define and explore novel classes of sets termed pre star generalized closed sets ($P^* g - closed$), pre star generalized open sets ($P^* g - open$) within the context of topological spaces. The relationship of this set with other closed sets has been proven, and its fundamental properties such as union, intersection, and containment have been established. We have also demonstrated that pre star generalized closed set (P*g- closed) in any given set (let it be X) remains a pre star generalized closed set (P*g-closed) in any of its subset $Y \subseteq X$. Moreover, we have proven the fundamental properties of pre star generalized open set (P*g – open). In future studies, we aim to extend this research to introduce a new operator similar to the one currently under investigation, sharing its topological characteristics.

Keywords: P − closet, P − open, $P^* g - closed$, $P^* g - open$.

http://www.doi.org/10.62341/nafa3003

المجموعات المغلقة المعممة قبل النجمية في الفضاءات التوبولوجية

نادية أحمد محمد التومي، فاطمة أحمد محمد التومي
جامعة الزاوية – كلية التربية الزاوية – ليبيا
na.altoumi@zu.edu.ly

الملخص:

تلعب المجموعات المغلقة دورًا أساسيًا في الفضاءات التبولوجية. ومن الجدير بالذكر أنه يمكن حتى وصف تبولوجيا على مجموعة من خلال تحديد خصائص مجموعاتها المغلقة. في عام 1970، قدم ليفين مفهوم المجموعات المغلقة المعممة، والتي تُعرّف كالتالي: تعتبر المجموعة الفرعية S من الفضاء التبولوجي X مغلقة معممة إذا كان cl(A)⊆U عندما تكون A⊆U و U مجموعة مفتوحة. في هذا البحث، نقدم وندرس فئات جديدة من المجموعات تسمي المجموعات المغلقة المعممة قبل النجمية والمجموعات المفتوحة المعممة قبل النجمية في الفضاءات التوبولوجية. وقد تمت برهنة علاقة هذه المجموعة بالمجموعات المغلقة الأخرى وكذلك تم إثبات خصائصها الأساسية كالاتحاد والتقاطع والاحتواء وأيضا تطرقنا الي برهنة أن المجموعة المغلقة المعممة قبل النجمية الجزئية من أي مجموعة ولتكن X فهي أيضا مجموعة مغلقة معممة قبل النجمية في أي مجموعة جزئية منها $Y \subseteq X$ ، علاوة علي ذلك أثبتنا الخواص الأساسية للمجموعات المفتوحة المعممة قبل النجمية. وسوف نقوم في دراسات قادمة بتوسيع هذه الدراسة بحيث يمكن إدخال مؤثر جديد مشابه للمؤثر الذي قمنا بدراسته حاليا من حيث الخواص التوبولوجيا.

الكلمات المفتاحية: مجموعة مغلقة مسبقاً – مجموعة مفتوحة مسبقاً – المجموعات المغلقة المعممة قبل النجومية – المجموعات المفتوحة المعممة قبل النجومية

1. INTRODUCTION

In 1970, Levine [1] introduced the concept of generalized closed set (g-closed). These sets have since revealed numerous new properties of topological spaces, prompting extensive research by many scholars in recent years. Later, in (1982) A. S. Mashor, M.E Abd El-Monsef and S. N. Ei-Deeb [2] proposed the idea of pre-open set (p-open) and explored their properties within the framework of topology. In (1996) H. Maki, R. J. Umehaea and T. Noiri [3] introduced a novel type of generalized closed sets in topological space, termed pre generalized closed sets (Pg- closed).

http://www.doi.org/10.62341/nafa3003

This paper aim to further the study of P*g-closed sets thereby contributing new insights and concepts to the field of topology through both analytical and research-based approaches. The paper presents the notion of P*g- closed sets and provides various characterizations of these sets. Throughout this paper, (X,τ) represent the non-empty topological space on which no separation axioms are assumed, unless otherwise mentioned.
The current study focuses on the concept of pre star generalized closed sets (p*g-closed) and explores their fundamental properties within the framework of topological space. The investigation aims to provide analysis of these sets and their characteristics, contributing to a deeper understanding of their role in topology.

2. PRELIMINARIES

Let (X,τ) be a topological space [4]. If A is a non-empty subset of (X,τ) then:

- The closure of A, denoted by $cl(A)$, is the intersection of all closed sets containing A.
- The interior of A, denoted by $int(A)$, is the union of all open sets contained in A.

We recall the following definitions:

2.1 Definition [1,5]

Let A be a subset of a topological space (X,τ). Then:

1) A is called a generalized closed set (briefly g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X.

2) A is called a generalized open set (briefly g-open) if its complement is g-closed set in X.

2.2 Definition [6,7]

Suppose that (X,τ) is a topological space and $A \subseteq U$. Then:

1)The τ-closur of A denoted by $cl(A)$, is defined as:
$cl(A) = \bigcap\{F \subseteq U : A \subseteq F \ and \ F \in \mathcal{F}\}$.

2) The τ-interior of A, denoted by $int(A)$, is defined as:
$int(A) = \bigcup\{G \subseteq U : G \subseteq A \ and \ G \in \tau\}$.

http://www.doi.org/10.62341/nafa3003

2.3 Definition [8, 9]

Suppose that (X, τ) is a topological space. Then:

1) A subset $A \subseteq X$ is called pre-closed set (briefly P-closed) if $cl\big(int(A)\big) \subseteq A$.

2) A subset $A \subseteq X$ is called pre- open set (briefly P-open) if $A \subseteq int\big(cl(A)\big)$.

2.4 Definition [10]

Let A be a subset of a space X. The pre-interior of A, denoted by (P-$int(A)$), is the union of al pre-open sets (P-open) contained in A.

2.5 Definition [11]

Let A be a subset of a space X. The pre-closed sets of A, denoted by $\big(Pcl(A)\big)$, is the intersection of all pre-closed sets (P-closed) containing A.

2.6 Definition [3,12]

A subset A of a topological space X is called:

1) A generalized pre closed set (briefly -closed) if $Pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X.

2) A generalized pre-open set (briefly gP-open) if its complement is generalized pre closed (gP-closed) set in X.

2.7 Definition [3,13]

Suppose that (X, τ) is a topological space. A subset $A \subseteq X$ is said to be:

1) A Pre generalized closed set (briefly Pg-closed) if $Pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is pre-open set (P-open) in X.

2) A Pre generalized open set (briefly Pg-open) if its complement is pre generalized closed set (Pg-closed) in X.

http://www.doi.org/10.62341/nafa3003

3. PRE STAR GENERALIZED CLOSED SETS($\boldsymbol{P^{*}g-CLOSED}$)

In this section, we introduce the concept of pre star generalized closed sets ($P^{*}g-closed$) in topological spaces.

3.1 Definition

A set A of a topological spaces (X,τ) is called pre star generalized closed set ($P^{*}g-closed$) if $Pcl(A)\subseteq U$ whenever $A\subseteq U$ and U is $Pg-$open.

3.2 Example

Let$X=\{a,b,c\},\tau=\{\emptyset,X,\{a\},\{c\},\{a,c\},\{b,c\}\}$. Then $\{a,c\}$ is $P^{*}g-closed$ set in X.

Next, we establish the characterization of pre star generalized closed sets ($P^{*}g-closed$) using different generalization of closed sets and pre-open sets in the following theorem.

3.3 Theorem

Let (X,τ) be a topological space and $A\subseteq X$. Then the following statements hold:

a) If A is a generalized closed set (g– closed), then A is a pre star generalized closed set($P^{*}g-closed$).

b) If A is pre generalized closed set (Pg– closed), then A is pre star generalized closed set ($P^{*}g-closed$).

Proof

a)- Assume that A is generalized closed set(g-closed). Let $A\subseteq U$, where U is open in X. Since every open set is pre open set (P-open) in X, U is also pre open (P-open) in X. Moreover, every pre open set is pre generalized open set (Pg-open). Therefore, $Pcl(A)\subseteq U$ which implies that A is pre star generalized closed set ($P^{*}g-closed$).

b) Suppose that A is pre generalized closed set ($Pg-closed$) in X. Let $A\subseteq U$ and U is pre-open(P-open) in X. Since every pre-open set (P-open) is pre generalized open set (Pg-open) in X, we have U is pre generalized open (Pg- open) in X. Then,

http://www.doi.org/10.62341/nafa3003

$Pcl(A) \subseteq U$ since A is pre generalized closed set $(Pg - closed)$. Since $Pcl(A) \subseteq cl(A)$, we have pcl(A) ⊆ U. Consequently, A is pre star generalized closed set $(P^*g - closed)$.

3.4 Remark

In general, a closed set in X is not necessarily a pre star generalized closed set $(P^*g - closed)$ in X, as illustrated in the following example.

3.5 Example

In example 3.2, the set $\{b\}$ is closed in X, but it does not qualify as a pre star generalized closed set $(P^*g - \text{closed})$ in X.

3.6 Theorem

If A is pre star generalized closed set $(P^*g - closed)$ in X and $A \subseteq B \subseteq Pcl(A)$, then B is also pre star generalized closed set $(P^*g - closed)$.

Proof

Assume A is a pre star generalized closed set $(P^*g - closed)$ in X and $A \subseteq B \subseteq Pcl(A)$. Let $B \subseteq U$ where U is pre open set in X. Since $A \subseteq B$ and $B \subseteq U$, it follows that $A \subseteq U$. Because A is pre star generalized closed set $(P^*g - closed)$, $Pcl(A) \subseteq U$. Given $B \subseteq Pcl(A)$, we have $Pcl(B) \subseteq Pcl(A) \subseteq U$. Therefore, B is pre star generalized closed set $(P^*g - closed)$.

3.7 Theorem

Union of two pre star generalized closed sets $(P^*g - closed)$ in X is a pre star generalized closed set $(P^*g - closed)$ in X.

Proof

Let A and B be two pre star generalized closed sets $(P^*g - closed)$. Let U be pre generalized open set (Pg-open) in X and $A \subseteq U, B \subseteq U$ then $(A \cup B) \subseteq U$. Then $Pcl(A) \subseteq U$ & $Pcl(B) \subseteq U, A \subseteq U, B \subseteq U$. Since A and B are pre star generalized closed sets $P^*g - closed$ and U is Pre generalized open set (Pg-open) in X, we have $Pcl(A) \subseteq U$ and $Pcl(B) \subseteq U$. Since $A \cup B \subseteq U, Pcl(A) \subseteq U, Pcl(B) \subseteq U$, so that $Pcl(A \cup B) \subseteq \left(Pcl(A) \cup Pcl(B)\right) \subseteq U$. This completes the proof.

http://www.doi.org/10.62341/nafa3003

3.8 Theorem

Intersection of two subsets of pre star generalized closed sets $(P^{*}g-closed)$in X is a pre star generalized closed set ($P^{*}g-closed$) in X.

Proof

Let A and B be two pre star generalized closed sets $(P^{*}g-closed)$. Suppose U is a pre generalized open set (Pg-open) in X such that $A\subseteq U, B\subseteq U$. Then $Pcl(A\cap B)\subseteq U$. Hence $A\cap B$ is a pre star generalized closed set $(P^{*}g-closed)$ in X.

3.9 Theorem

Let $A\subseteq Y\subseteq X$. If A is pre star generalized closed set$(P^{*}g-closed)$ in X, then A is also a pre star generalized closed set $(P^{*}g-closed)$ relative to Y.

Proof

Assume $A\subseteq Y\cap U$ where U is an open in X and thus pre generalized open in X. Since A is pre star generalized closed set $(P^{*}g-closed)$ in X, $A\subseteq U$ which implies $Pcl(A)\subseteq U$. That is $Y\cap Pcl(A)\subseteq Y\cap U$, where $Y\cap Pcl(A)$ is the pre closure of A. Thus A ispre star generalized closed set $(P^{*}g-closed)$ relative to Y.

4. PRE STAR GENERALIZED OPEN SETS $(\mathbf{P^{*}g-OPEN})$

In this section, we introduce and analyze the concept of pre star generalized open sets $(briefly\ P^{*}g-\text{open})$ within a topological space X.

4.1 Definition

A subset A of X is termed a pre star generalized open set$(briefly\ P^{*}g-\text{open})$ if and only if A^{c} is pre star generalized closed set $(P^{*}g-closed)$in X.

4.2 Example

Referring to example 3.2, the set $\{b\}$ is a pre star generalized open set $(P^{*}g-\text{open})$ in X.

4.3 Theorem

Suppose that (X,τ) is a topological space and $A\subseteq X$. The following statements hold.

http://www.doi.org/10.62341/nafa3003

a)- If A is g – open set, then A is pre star generalized open set($P^*g - open$).
b)- If $A$is pre generalized open set (pg – open), then A is pre star generalized open set ($P^*g - open$).

4.4 Theorem

If A and $B$are pre star generalized open sets ($P^*g -$ open) then so is $A \cap B$.

Proof

Suppose that A and B are pre star generalized open sets ($P^*g -$ open) in a space X. Consequently, their A^c and B^c are pre star generalized closed sets ($P^*g -$ closed) in a space X. By Theorem (3.5) $A^c \cup B^c$ is also pre star generalized closed set ($P^*g -$ closed) in space X. Since $A^c \cup B^c = (A \cap B)^c$, it follows that $(A \cap B)^c$, is a pre star generalized closed set ($P^*g -$ closed) in space X. Therefore $A \cap B$ is pre star generalized open set ($P^*g -$ open) in X.

4.5 Theorem

If A is a pre star generalized open set ($P^*g -$ open) in X and$int(A) \subseteq B \subseteq A$, then B is pre star generalized open set ($P^*g -$ open).

Proof

Suppose A is pre star generalized open set ($P^*g -$ open) in X and $int(A) \subseteq B \subseteq A$, then $A^c \subseteq B^c \subseteq cl(A^c)$. Since A is pre star generalized open set ($P^*g -$ ope). Then A^C is pre star generalized closed set ($P^*g -$ closed) in space X. This implies that B^c is also pre star generalized closed set ($P^*g -$ closed), and thus B is pre star generalized open set ($P^*g -$ open) in space X.

4.6 Theorem

A set A is pre star generalized open set ($P^*g -$ open) in a space X if and only if for every pre generalized closed set ($Pg - closed$) F such that $F \subseteq A$, it holds that $F \subseteq Int(A)$.

Proof

Since set A is pre star generalized open set ($P^*g -$ open) in a space X, its A^c is pre star generalized closed set ($P^*g -$ closed) in X, For any pre generalized closed set ($Pg - closed$) F with $F \subseteq A$, F^c is pre generalized open set ($Pg -$ open) and $A^c \subseteq$

http://www.doi.org/10.62341/nafa3003

F^c. Therefore $cl\,(A^c) \subseteq (F^c)$. Noting that $cl(A^c) = \big(Int(A)\big)^c$, we have $\big(Int(A)\big)^c \subseteq F^c$, which implies $F \subseteq Int(A)$. This completes the proof

5. CONCLUSION

This paper introduces and examines the concepts of pre star generalized closed sets $(P^*g - closed)$ and pre star generalized open sets (P^*g − open). Within topological spaces, exploring their fundamental properties.

The pre star generalized closed sets $(P^*g - closed)$ can be used to derive a new homeomorphism, connectedness, compactness and new separation axioms. This concept can be extended to bitopological and fuzzy topological spaces.

6. REFERENCES

[1]- N. Lovine, “Generalized closed sets topology”. Rend. Circ. Math. Palermo, 19 (2) (1970) 89-96.

[2]- A.S. Mashor, M. E. Abd. El-Monsef and S. N. Ei-Deeb, “On Pre continous and weapre- continous mapping”, Proc. Math., Phys. Soc. Egypt, 53:47-53(1982).

[3]- H. Maki, J. Umehare and T. Nori, “Every topological space is pre-$T_{\frac{1}{2}}$”, Mem. Fac. So Kochi Univ. Math., 17:32-42(1996).

[4]- W. Dunham, “A new closure operator for non-T_1 topologies”, Kyungpook Math. J. 22:55-60 (1982).

[5]- K. Chandrasekhara Rao and K. Joseph. “Semi star generalized closed sets”. Bull Pure ApplSci 19E(No2):281-290(2000).

[6]- P. L. Meenakshi."J-closed sets in topological spaces". Journal of Emerging Technologies and innovative Research. Vol. 6. No.5, PP. 193-201., 2019.

[7]- Kelley, J.: “*General topology*”, Van NostrandCompany, (1955).

[8]- M. Pauline Mary Helen, A. Kulandhai Theresa, (gsp)*- closed sets in topological space, IJMTT 6 (2014).

[9]- Mashhour, A. S.; Abd El-Monsef, M.E.; EL-Deeb, S.N.: “on pre continuous and weak pre continuous mappings proc”. Math. Phys. Soc. Egypt, 53:47-53(1982).

[10]- A. S. Mashor, M. E. Abd. El-Monsef and I. A. Hasanein, “On pre topological Spaces”,Bull. Math. R. S. Roumanie, 28(76):39-45 (1984).

[11]- A. A. El-Atik, “A study on some types of mappings on

http://www.doi.org/10.62341/nafa3003

topological space", Master's Thesis, Tanta University, Egypt, (1997).

[12]- D. Narashimhan and J. Jayanthi, On gp**- closed set in topological spaces, International journal of pure and applied Mathematics, volume 119 no.6.2018.

[13]- P. Sivagami, M. Padmavathi, G. H. S. Annam, μ-pre*-closed sets in generalized topological spaces, Malaya J. Mat. 8 (2020), 1331-1336.